

ELECTROMAGNETIC NON-CONTACT GEARS: PRELUDE

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We calculate the lateral Lifshitz force between corrugated dielectric slabs of finite thickness. Taking the thickness of the plates to infinity leads us to the lateral Lifshitz force between corrugated dielectric surfaces of infinite extent. Taking the dielectric constant to infinity leads us to the conductor limit which has been evaluated earlier in the literature.

1. Introduction

In past decade significant attention has been given to evaluation of the lateral force between corrugated surfaces (for example see Ref. 1–5 and references there-in). In an earlier work we calculated the contribution of the next-to-leading order to the lateral Casimir force between two corrugated semi-transparent δ -function plates interacting with a scalar field,⁴ and the leading order contribution for the case of two concentric semi-transparent corrugated cylinders⁵ using the multiple scattering formalism (see Ref. 6,7 and references there-in). We observed that including the next-to-leading order contribution significantly reduced the deviation from the exact result in the case of weak coupling. Comparison with experiments requires the analogous calculation for the electromagnetic case. Here we present preliminary results of our ongoing work on the evaluation of the lateral Lifshitz force between two corrugated dielectric (non-magnetic) slabs of finite thickness

interacting through the electromagnetic field (see Fig. 1).

From the general result it is easy to take various limiting cases. Taking the thickness of the dielectric slabs to infinity leads us to the lateral Lifshitz force between dielectric slabs of infinite extent. The lateral Casimir force between corrugated conductors was evaluated by Emig et al.¹ In our situation this is achieved by taking the dielectric constants $\varepsilon_i \rightarrow \infty$. Our results agree with the results in Emig et al.¹ Taking the thin-plate approximation based on the plasma model we have calculated the lateral force between corrugated plasma sheets. Our goal is to extend these results to next-to-leading order. Most of these will appear in a forthcoming paper.

2. Interaction energy

We consider two dielectric slabs of infinite extent in x - y plane, which have corrugations in y -direction, as described in Fig. 1. We describe the dielectric slabs by the potentials

$$V_i(z, y) = (\varepsilon_i - 1) [\theta(z - a_i - h_i(y)) - \theta(z - b_i - h_i(y))], \quad (1)$$

where $i = 1, 2$, designates the individual dielectric slabs. $\theta(z)$ is the Heaviside theta function defined to equal 1 for $z > 0$, and 0 when $z < 0$. $h_i(y)$ describes the corrugations on the surface of the slabs. We define the thickness of the individual slabs as $d_i = b_i - a_i$, such that $a = a_2 - b_1 > 0$ represents the distance between the slabs. The permittivities of the slabs are represented by ε_i .

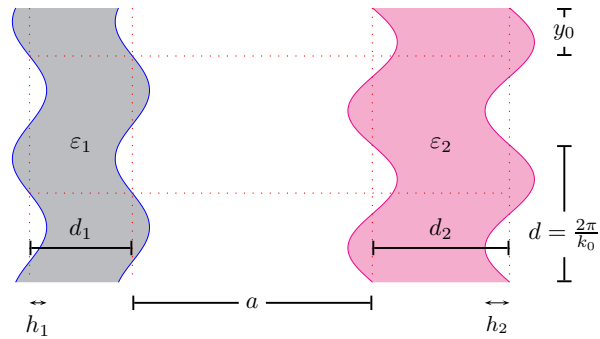


Fig. 1. Parallel dielectric slabs with sinusoidal corrugations.

Using the multiple scattering formalism for the case of the electromag-

netic field^{8,9} based on Schwinger's Green's dyadic formalism¹⁰ and following the formalism described in Gears-I⁴ we can obtain the contribution to the interaction energy between the two slabs in leading order in the corrugation amplitudes to be

$$E_{12}^{(2)} = \frac{i}{2} \int \frac{d\omega}{2\pi} \text{Tr} \left[\mathbf{\Gamma}^{(0)} \Delta V_1^{(1)} \cdot \mathbf{\Gamma}^{(0)} \Delta V_2^{(1)} \right], \quad (2)$$

where $\Delta V_i^{(1)}$ are the leading order contributions in the potentials due to the presence of corrugations. In particular, we have

$$\Delta V_i^{(1)}(z, y) = -h_i(y) (\varepsilon_i - 1) [\delta(z - a_i) - \delta(z - b_i)]. \quad (3)$$

Note that $V_i^{(0)}$ describes the potential for the case when the corrugations are absent and represent the background in the formalism. $\mathbf{\Gamma}^{(0)} = \mathbf{\Gamma}^{(0)}(\mathbf{x}, \mathbf{x}'; \omega)$ is the Green's dyadic in the presence of background potential $V_i^{(0)}$ and satisfies

$$\left[-\frac{1}{\omega^2} \nabla \times \nabla \times + \mathbf{1} + V_1^{(0)} + V_2^{(0)} \right] \cdot \mathbf{\Gamma}^{(0)} = -\mathbf{1}. \quad (4)$$

The corresponding reduced Green's dyadic $\gamma^{(0)}(z, z'; k_x, k_y, \omega)$ is defined by Fourier transforming in the transverse variables as

$$\mathbf{\Gamma}^{(0)}(\mathbf{x}, \mathbf{x}'; \omega) = \int \frac{dk_x}{2\pi} \frac{dk_y}{2\pi} e^{ik_x(x-x')} e^{ik_y(y-y')} \gamma^{(0)}(z, z'; k_x, k_y, \omega). \quad (5)$$

Using the fact that our system is translationally invariant in the x -direction, we can write

$$\frac{E_{12}^{(2)}}{L_x} = \int_{-\infty}^{\infty} \frac{dk_y}{2\pi} \int_{-\infty}^{\infty} \frac{dk'_y}{2\pi} \tilde{h}_1(k_y - k'_y) \tilde{h}_2(k'_y - k_y) L^{(2)}(k_y, k'_y), \quad (6)$$

where L_x is the length in the x -direction and $\tilde{h}_i(k_y)$ are the Fourier transforms of the functions $h_i(y)$ describing the corrugations. The kernel $L^{(2)}(k_y, k'_y)$ is given by

$$L^{(2)}(k_y, k'_y) = -\frac{1}{2} \int \frac{d\zeta}{2\pi} \int \frac{dk_x}{2\pi} I^{(2)}(k_x, \zeta, k_y, k'_y), \quad (7)$$

where

$$\begin{aligned} I^{(2)}(k_x, \zeta, k_y, k'_y) &= (\varepsilon_a - 1) (\varepsilon_b - 1) \\ &\times \left[\gamma^{(0)}(a_2, a_1; k_x, k_y, \omega) \cdot \gamma^{(0)}(a_1, a_2; k_x, k'_y, \omega) \right. \\ &\quad - \gamma^{(0)}(b_2, a_1; k_x, k_y, \omega) \cdot \gamma^{(0)}(a_1, b_2; k_x, k'_y, \omega) \\ &\quad - \gamma^{(0)}(a_2, b_1; k_x, k_y, \omega) \cdot \gamma^{(0)}(b_1, a_2; k_x, k'_y, \omega) \\ &\quad \left. + \gamma^{(0)}(b_2, b_1; k_x, k_y, \omega) \cdot \gamma^{(0)}(b_1, b_2; k_x, k'_y, \omega) \right], \quad (8) \end{aligned}$$

where the reduced Green's dyadics are evaluated after solving Eq. (4). We note that $\gamma^{(0)}(z, z'; k_x, k_y, \omega) = \gamma^{(0)\dagger}(z', z; k_x, k_y, \omega)$. Our task reduces to evaluating the reduced Green's dyadic in the presence of the background. The details of this evaluation will be described in the forthcoming paper.

2.1. Evaluation of the reduced Green's dyadic

The Green's dyadic satisfies Eq. (4) whose solution can be determined by following the procedure described in Schwinger et al.¹⁰ The expression for the reduced Green's dyadic

$$\gamma^{(0)}(z, z'; k, 0, \zeta) = \begin{bmatrix} \frac{1}{\varepsilon(z)} \frac{\partial}{\partial z} \frac{1}{\varepsilon(z')} \frac{\partial}{\partial z'} g^H & 0 & \frac{ik}{\varepsilon(z')} \frac{1}{\varepsilon(z)} \frac{\partial}{\partial z} g^H \\ 0 & -\zeta^2 g^E & 0 \\ -\frac{ik}{\varepsilon(z)} \frac{1}{\varepsilon(z')} \frac{\partial}{\partial z'} g^H & 0 & \frac{k^2}{\varepsilon(z)\varepsilon(z')} g^H \end{bmatrix} \quad (9)$$

is given in terms of the electric and magnetic Green's functions* $g^E(z, z')$ and $g^H(z, z')$, which satisfy the following differential equations:

$$-\left[\frac{\partial^2}{\partial z^2} - k^2 - \zeta^2 \varepsilon(z) \right] g^E(z, z') = \delta(z - z'), \quad (10)$$

$$-\left[\frac{\partial}{\partial z} \frac{1}{\varepsilon(z)} \frac{\partial}{\partial z} - \frac{k^2}{\varepsilon(z)} - \zeta^2 \right] g^H(z, z') = \delta(z - z'). \quad (11)$$

We have used the definitions $k^2 = k_x^2 + k_y^2$ and $\varepsilon(z) = 1 + V_1^{(0)}(z) + V_2^{(0)}(z)$.

The reduced Green's dyadic for arbitrary k_y is generated by the rotation

$$\gamma^{(0)}(z, z'; k_x, k_y, \zeta) = \mathbf{R} \cdot \gamma^{(0)}(z, z'; k, 0, \zeta) \cdot \mathbf{R}^T, \quad (12)$$

where

$$\mathbf{R} = \frac{1}{k} \begin{pmatrix} k_x & -k_y & 0 \\ k_y & k_x & 0 \\ 0 & 0 & k \end{pmatrix}. \quad (13)$$

We have dropped delta functions in Eq. (9) because they are evaluated at different points and thus do not contribute. We shall not present explicit solutions to the electric and magnetic Green's functions here which will be presented in our forthcoming paper.

*Here we use the notation in Schwinger et al.¹⁰ which was reversed in many of Milton's publications, for example in Milton's book.¹¹

2.2. Interaction energy for corrugated dielectric slabs

Using the solutions to the electric and magnetic Green's function in Eq. (9) we can evaluate $I^{(2)}(k_x, \zeta, k_y, k'_y)$ in Eq. (8) as

$$\begin{aligned} & -\frac{1}{k^2} \frac{1}{k'^2} \frac{1}{2\kappa} \frac{1}{2\kappa'} \left[\frac{1}{\Delta} \frac{1}{\Delta'} M(-\alpha_1, -\alpha'_1) M(-\alpha_2, -\alpha'_2) (k_x^2 + k_y k'_y)^2 \zeta^4 \right. \\ & + \frac{1}{\Delta} \frac{1}{\Delta'} M(-\alpha_1, \bar{\alpha}'_1) M(-\alpha_2, \bar{\alpha}'_2) k_x^2 (k_y - k'_y)^2 \zeta^2 \kappa'^2 \\ & + \frac{1}{\Delta} \frac{1}{\Delta'} M(\bar{\alpha}_1, -\alpha'_1) M(\bar{\alpha}_2, -\alpha'_2) k_x^2 (k_y - k'_y)^2 \zeta^2 \kappa^2 \\ & + \frac{1}{\Delta} \frac{1}{\Delta'} \left\{ M(\bar{\alpha}_1, \bar{\alpha}'_1) (k_x^2 + k_y k'_y) \kappa \kappa' + M(-\bar{\alpha}_1, -\bar{\alpha}'_1) k^2 k'^2 \frac{1}{\varepsilon_1} \right\} \\ & \times \left. \left\{ M(\bar{\alpha}_2, \bar{\alpha}'_2) (k_x^2 + k_y k'_y) \kappa \kappa' + M(-\bar{\alpha}_2, -\bar{\alpha}'_2) k^2 k'^2 \frac{1}{\varepsilon_2} \right\} \right], \quad (14) \end{aligned}$$

where

$$\Delta = [(1 - \alpha_1^2 e^{-2\kappa_1 d_1})(1 - \alpha_2^2 e^{-2\kappa_2 d_2}) e^{\kappa a} - \alpha_1 \alpha_2 (1 - e^{-2\kappa_1 d_1})(1 - e^{-2\kappa_2 d_2}) e^{-\kappa a}], \quad (15)$$

$$\begin{aligned} M(\alpha_i, \alpha'_i) &= (\varepsilon_i - 1) [(1 - \alpha_i^2) e^{-\kappa_i d_i} (1 - \alpha_i'^2) e^{-\kappa_i' d_i} \\ &\quad - (1 + \alpha_i)(1 - \alpha_i e^{-2\kappa_i d_i})(1 + \alpha_i')(1 - \alpha_i' e^{-2\kappa_i' d_i})], \quad (16) \end{aligned}$$

where $\kappa_i^2 = k^2 + \zeta^2 \varepsilon_i$, $\bar{\kappa}_i = \kappa_i / \varepsilon_i$, and $\alpha_i = (\kappa_i - \kappa) / (\kappa_i + \kappa)$. Quantities with primes are obtained by replacing $k_y \rightarrow k'_y$ everywhere, and quantities with bars are obtained by replacing κ_i with $\bar{\kappa}_i$ except in the exponentials.

2.3. Conductor limit

In the conductor limit ($\varepsilon_i \rightarrow \infty$) the above expression takes the form

$$I_{\varepsilon \rightarrow \infty}^{(2)}(\kappa, \kappa', k_y - k'_y) = -\frac{\kappa}{\sinh \kappa a} \frac{\kappa'}{\sinh \kappa' a} \left[1 + \frac{\{\kappa^2 + \kappa'^2 - (k_y - k'_y)^2\}^2}{4 \kappa^2 \kappa'^2} \right]. \quad (17)$$

For the case of sinusoidal corrugations described by $h_1(y) = h_1 \sin[k_0(y + y_0)]$ and $h_2(y) = h_2 \sin[k_0 y]$ the lateral force can be evaluated to be

$$F_{\varepsilon \rightarrow \infty}^{(2)} = 2k_0 a \sin(k_0 y_0) \left| F_{\text{Cas}}^{(0)} \right| \frac{h_1}{a} \frac{h_2}{a} A_{\varepsilon \rightarrow \infty}^{(1,1)}(k_0 a), \quad (18)$$

where

$$A_{\varepsilon \rightarrow \infty}^{(1,1)}(t_0) = \frac{15}{\pi^4} \int_{-\infty}^{\infty} dt \int_0^{\infty} \bar{s} d\bar{s} \frac{s}{\sinh s} \frac{s_+}{\sinh s_+} \left[\frac{1}{2} + \frac{(s^2 + s_+^2 - t_0^2)^2}{8 s^2 s_+^2} \right], \quad (19)$$

where $s^2 = \bar{s}^2 + t^2$ and $s_+^2 = \bar{s}^2 + (t + t_0)^2$. The first term in Eq. (19) corresponds to the Dirichlet scalar case,⁴ which here corresponds to the E mode (referred to in Ref. 1 as TM mode). We note that $A_{\varepsilon \rightarrow \infty}^{(1,1)}(0) = 1$. See Fig. 2 for the plot of $A_{\varepsilon \rightarrow \infty}^{(1,1)}(k_0 a)$ versus $k_0 a$. We observe that only in the PFA limit is the electromagnetic contribution twice that of the Dirichlet case, and in general the electromagnetic case is less than twice that of the Dirichlet case.

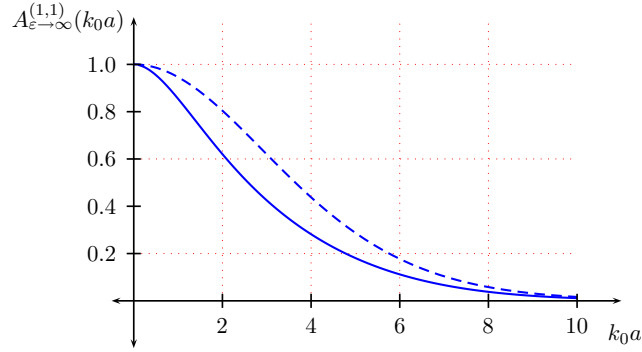


Fig. 2. Plot of $A_{\varepsilon \rightarrow \infty}^{(1,1)}(k_0 a)$ versus $k_0 a$. The dotted curve represents 2 times the Dirichlet case.

Since the above expression involves a convolution of two functions we can evaluate one of the integrals to get

$$A_{\varepsilon \rightarrow \infty}^{(1,1)}(t_0) = \frac{15}{4} \int_0^\infty du \frac{\sin(2t_0 u/\pi)}{(2t_0 u/\pi)} \left[\frac{\sinh^2 u}{\cosh^6 u} \left(\frac{7}{2} - \sinh^2 u \right) - \frac{1}{2} \left(\frac{2t_0}{\pi} \right)^2 \frac{\sinh^2 u}{\cosh^4 u} + \frac{1}{16} \left(\frac{2t_0}{\pi} \right)^4 \frac{\sinh^2 u}{\cosh^2 u} \right], \quad (20)$$

which reproduces the result in Emig et al¹ apart from an overall factor of 2, which presumably is a transcription error. Even though Eq. (20) involves only a single integral it turns out that the double integral representation in Eq. (19) is more useful for numerical evaluation because of the oscillatory nature of the function $\sin x/x$ in the former.

3. Conclusion

We have evaluated leading order contribution to the lateral Lifshitz force between two corrugated dielectric slabs. Taking the dielectric constants of the two bodies to infinity gives the lateral Casimir force between corrugated conductors. We shall extend these results to next-to-leading order contribution for a better comparison with experiments in future publication as well as include various other limiting cases, which can be readily obtained from Eq. (14).

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